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Rithmomachia: An Academic Proposal of Rules

Rié Durnil, Tomás Guardia

Abstract

This paper is a proposal of a set of rules for the medieval math game of Rithmomachia. The set we introduce results from the reading and interpretation of the rules from several medieval manuals. The primary source is the manual of Ralph Lever and William Fulke from 1556 that is found in the appendix of Moyer [4]. As a result of testing and studying the rules, we introduce a new rule that we coin the *Innocent Delivery*. No rules of Rithmomachia are definitive due to the different ways of capturing pieces as well as the game's ending. We offer this proposal a starting point for further academic discussion and testing of a future set of simplified rules in the hopes of a definitive set of rules for future Rithmomachia players.

Key words and phrases: Board Games, Rithmomachia, Rules.

Ritmomaquia: Una Propuesta Académica de Reglas

Resumen

Este artículo es una propuesta de un conjunto de reglas para el juego matemático medieval Ritmomaquia. El conjunto que introducimos resulta de la lectura e interpretación de las reglas de varios manuales medievales. La fuente primaria es el manual de Ralph Lever y William Fulke de 1556, que se encuentra en el apéndice de Moyer [4]. Como resultado de la prueba y el estudio de las reglas, introducimos una nueva regla que llamamos la *Entrega Inocente*. Ninguna regla de Ritmomaquia es definitiva debido a las diferentes maneras de capturar piezas así como al final del juego. Nosotros ofrecemos esta propuesta como un punto de partida para posteriores discusiones académicas y puesta a prueba de un futuro conjuntos simplificado de reglas, que esperamos sea un conjunto definitivo de reglas para futuros jugadores de Ritmomaquia.

Palabras y frases clave: Juegos de tablero, Ritmomaquia, Reglas.

1 Introduction

Rithmomachia is a mathematical strategy board game that was popular in monastic schools and medieval universities. After the Renaissance, the game gradually disappeared, leaving only a few old manuals as reference. Unfortunately, the rules, setup, and plays described by these manuals differ significantly, and with no definitive set attributed as the original, there is also no consistent set of rules among modern Rithmomachia players. Discussion will be necessary in establishing a final set of rules among players worldwide in an attempt to make the rules both historically accurate and most player-friendly. As an attempt to begin the academic debate on the rules, this paper will introduce a set of rules that have been practiced, tested and modified by real players since 2012, first in Venezuela, then in the United States.

2 Origin

Many sources attribute the origin of Rithmomachia to Julius Caesar, Alexander the Great, and the first Pythagoreans[1]. However, there are no definitive sources to back up these claims. The earliest source with an elementary description of Rithmomachia was written by Asilo von Würzburg[2]. While Würzburg claims to have invented the game, there is no evidence, so we just credit him as the earliest source.

Rithmomachia was an intellectual game that developed in the elite of the Cathedral Schools and Universities of the Middle Ages, with the intent to teach and exercise the study of Boethian Mathematics[3]. The main references coming from the Renaissance are Claude de Boissière from France, Ralph Lever and William Fulke from England, and Bennedetto Varchi, Carlo Strozzi, and Francesco Barozzi from Italy[4]. For a detailed history of the rise and fall of Rithmomachia, see Moyer [4].

The decentralized medieval society created the main problem we have in the present day with so much variation in rules among players. As Rithmomachia spread across medieval society, each author created their own version. There are many different board sizes, ways to move the pieces, and several

[1]Moyer [4], page 20.
[2]Moyer [4], page 20.
[3]Moyer [4], page 2.
[4]Moyer [4]. Chapter 4, pages 85-122.

styles of captures. The rules we propose in this paper follow the version of Ralph Lever and William Fulke from 1563 found in the appendix of [4], and include a new rule, the *Innocent Delivery*.

3 Board

While different versions of the board have been used at other times in the history of Rithmomachia, the standard board is sixteen squares long and eight squares wide. Its setup is shown in the figure below.

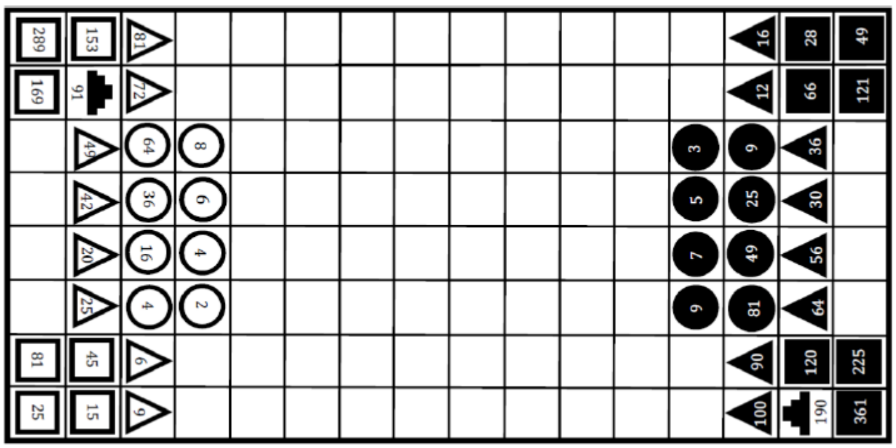


Figure 1: Starting Positions of the Pieces

4 Pieces and Movement

In Rithmomachia, there are three different shapes of pieces: circles, triangles, and squares. Each shape has a different movement. As seen in the setup, each player has an equal amount of each kind of shape. Aside from circles, pieces have two kinds of movement, regular and irregular. Regular movements occur in a straight line, horizontally and vertically, while irregular movements add a perpendicular shift. Only regular movements can capture the opponent's pieces, but irregular movements are extremely useful for escaping from possible attacks. The movements of each kind of piece will be explained in the following sections, however it is important to note the geometric significance of the maximum number of spaces each shape can move. If we assume that a circle only has one side, the maximum number of spaces each shape can move corresponds to the number of sides it has. So, while other sets of rules may differ, in our rules, the circle can only ever move one space, the triangle can move three spaces at most, and the square can move four spaces at most.

4.1 Circles

Circular pieces always move one space diagonally in any direction. These moves are limited by the bounds of the board and any pieces that may be in the spaces surrounding.

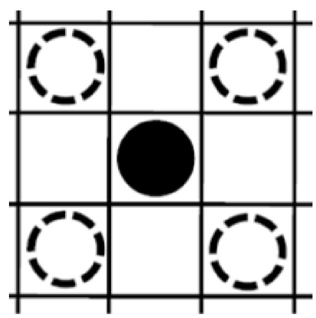


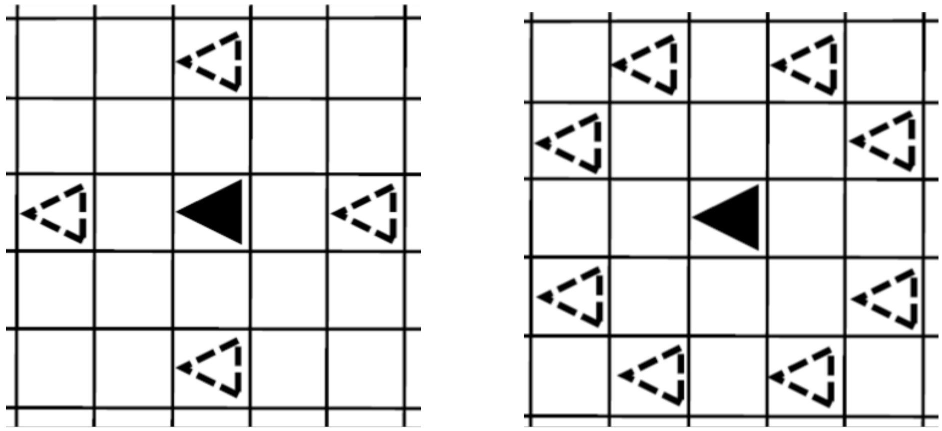
Figure 2: Movement of Circular Pieces

4.2 Triangles

The regular movement for a triangular piece is two spaces in any horizontal or vertical direction, though this movement may not jump over other pieces. The irregular movement is similar to that of a knight in chess – two spaces horizontally or vertically plus one space perpendicular. This movement is allowed to jump over other pieces.

4.3 Squares

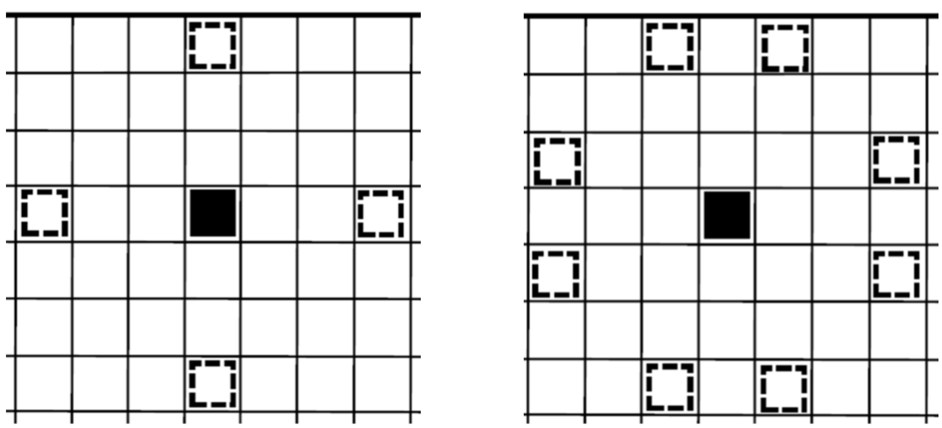
The regular movement for a square piece is three spaces in any horizontal or vertical direction, again with no jumps. The irregular movement is



(a) Regular Movement (b) Irregular Movement

Figure 3: Movements of Triangular Pieces

three spaces horizontally or vertically plus one space perpendicular. This movement can jump over other pieces as well.



(a) Regular Movement (b) Irregular Movement

Figure 4: Movements of Square Pieces

4.4 Pyramids

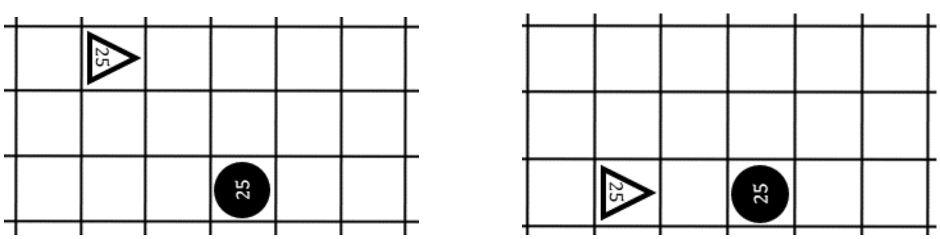
Each player has one pyramid consisting of consecutive squared numbers. For the white pieces, the pyramid begins with the square base of 36, then stacked on top is the square 25, the triangle 16, the triangle 9, the circle 4, and finally the circle 1. In some sets of the game, the top piece with the value of one is shaped as a cone, because of the Greek concept of the *μονάς* (monad), in which one is not a number but instead the unit by which all other numbers are made, *a unit is that by virtue of which each of existing things is called one*.[5] The black pyramid consists of the square base 64, then the square 49, the triangle 36, the triangle 25, and then finally the circle 16. Pyramids, because they consist of every type of piece, can move and capture as a square, a triangle, or a circle.

5 Captures

There are many different types of captures in Rithmomachia. Captures result in the removal of the captured opponent's piece from the board.

5.1 Captures by Encounter

The first and simplest kind of capture involves two opposing pieces of the same value. When by a regular movement, a piece becomes one more regular movement away from the opponent's piece of the same value, the opposing piece has been captured. As mentioned previously, irregular movements may not be used to get to this position.



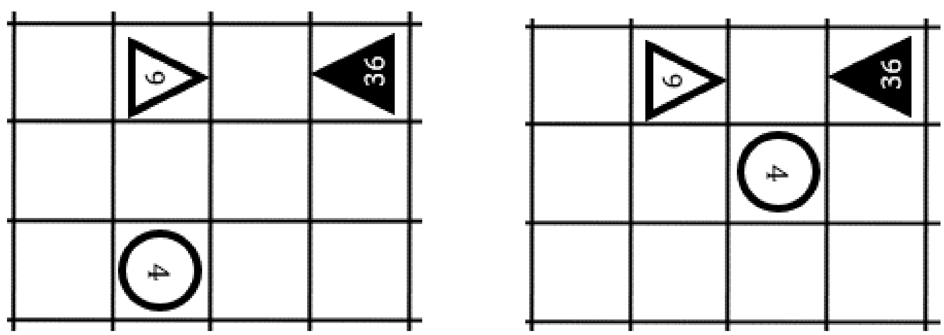
(a) Before (b) After

Figure 5: Example of Capture by Encounter: The white triangle 25 captures the black circle 25.

[5]For more details on the definition of the monad and the Greek concept of numbers, see Heath [2] pg. 69.

5.2 Captures by Ambush

Captures by ambush happen using two pieces to capture an opponent's piece. The values of the three pieces at play must be related through addition, subtraction, multiplication, or division. Essentially, the two captruing pieces must add, subtract, multiply, or divide with each other to get the value of the opponent's piece to be captured. Both capturing pieces must become one regular movement away from the opponent's piece through regular movements in order to legally make the capture.

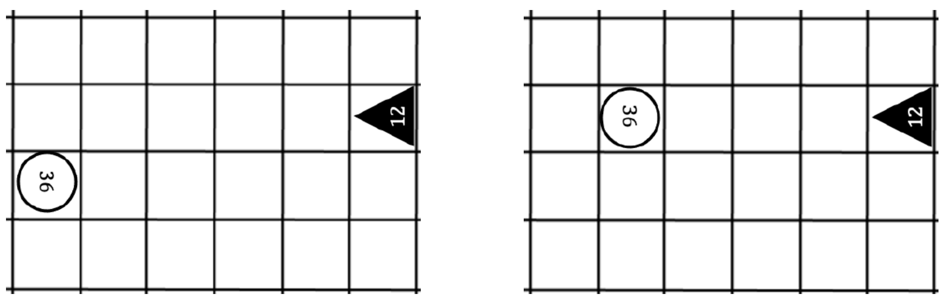


(a) Before (b) After

Figure 6: Example of Capture by Ambush: The white triangle 9 and white circle 4 capture the black triangle 36 since $9 * 4 = 36$.

5.3 Captures by Assault

One of the most common ways to capture, especially among newer players of Rithmomachia, is called Capturing by Assault. Captures by Assault use multiplication and division among one of each player's pieces and the number of spaces between them. Opposing pieces must be directly horizontal, vertical, or diagonal from each other at the time of the capture, and once again, to get in this position, only regular movements can be used – an irregular movement would make the capture invalid. As an example, if the capturing piece is a four, and it moves regularly such that it is directly vertical, four spaces away from the opponent's sixteen, the four has successfully captured the opponent's sixteen piece because four (the capturing piece) times four (the number of spaces between the pieces) is sixteen (the opponent's piece). The sixteen may also capture the four in this same way, because sixteen (the capturing piece) divided by four (the number of spaces between the pieces) equals four (the opponent's piece). Only multiplication and division can be used in these captures – addition and subtraction would not be valid.

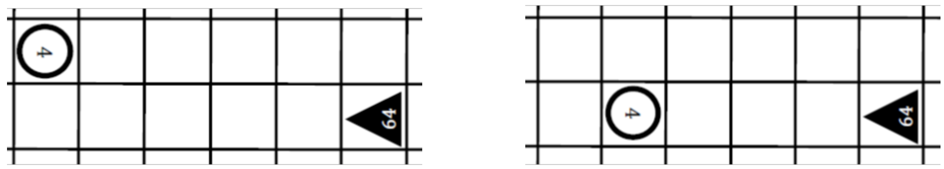


(a) Before (b) After

Figure 7: Example of Capture by Assault: The white circle 36 captures the black triangle 12 from 3 spaces away since $\frac{36}{3} = 12$.

5.4 Captures by Power and Root

Captures by Power and Root are similar in concept to Captures by Assault. However, instead of using multiplication or division with piece values and the number of spaces between them to capture an opponent's piece, powers and roots are used. Again, a regular movement must be used to get into a position horizontally, vertically, or diagonally the correct number of spaces away from the opponent's piece. An example of this would be a capturing piece of value five could capture the opponent's twenty-five piece if it became two spaces away from their piece through a regular movement, since five to the power of two equals twenty-five. The twenty-five could also capture the five in this way, since the square root of twenty-five equals five. Of course, higher exponents and roots may also be used given that the number of spaces also increases.



(a) Before (b) After

Figure 8: Example of Capture by Power and Root: The white circle 4 captures the black triangle 64 from 3 spaces away since $4^3 = 64$

5.5 Captures by Progression

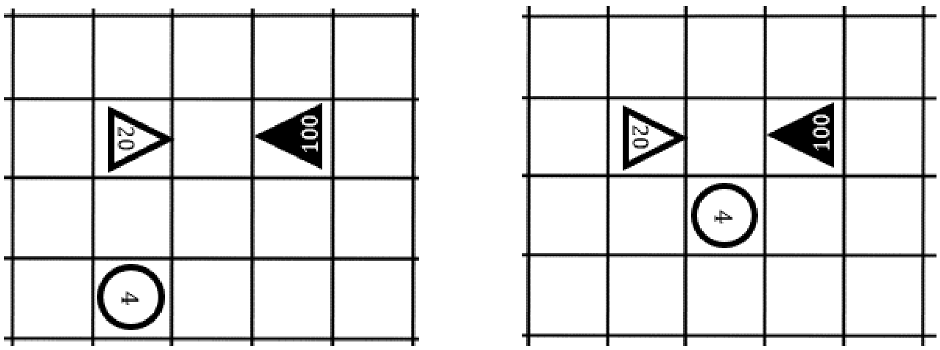
Captures by Progression take place when using arithmetic, geometric, or harmonic progressions. Each kind of progression requires two capturing pieces and one of the opponent's pieces.

Arithmetic progressions occur when the difference between the values of all three pieces is the same. In other words, the difference between the lowest and the middle valued piece is the same as the difference between the middle and the highest valued piece. As an example, the pieces four, seven, and ten form an arithmetic progression because the difference between four and seven is three, and the difference between seven and ten is also three.

Geometric progressions are similar to arithmetic progressions except instead of being based around addition and subtraction, multiplication and division are utilized. All three numbers in the sequence have a common multiplier, and the previous number must be multiplied by this common multiplier to get the next value. For example, the pieces three, nine, and twenty-seven are in geometric progression because three times three is nine, and nine times three is twenty-seven.

Harmonic progression is the most difficult of the progressions. Three numbers, a, b, and c, are in harmonic progression if and only if $b = \frac{2ac}{a+c}$. Another way to think about this is that numbers are in harmonic progression if their reciprocals are in arithmetic progression. Take the example of two, three, and six. Using the formula, we see that $3 = \frac{2(2)(6)}{2+6}$, which is in fact true. By taking the reciprocals, $\frac{1}{2}, \frac{1}{3}$, and $\frac{1}{6}$, we can also see that these numbers are in harmonic progression because for each reciprocal, $\frac{1}{6}$ is subtracted to get to the next value.

Capturing by Progression is similar in execution to Capturing by Ambush. In order to capture, through regular movements, two capturing pieces must each become one regular movement away from the opponent's piece that completes the progression. The order of the three values does not matter; in order to capture, the three pieces must simply be in a progression in any arrangement.



(a) Before (b) After

Figure 9: Example of Capture by (Geometric) Progression: The white circle 4 and the white triangle 20 capture the black triangle 100 since 4, 20, and 100 form a geometric progression

5.6 Captures by Surrounding

Captures by Surrounding occur when one of the opponent's pieces cannot make a movement, regular or irregular, in any direction due to the placement of the capturing pieces. This can take place by an edge of the board or entirely by pieces surrounding it.

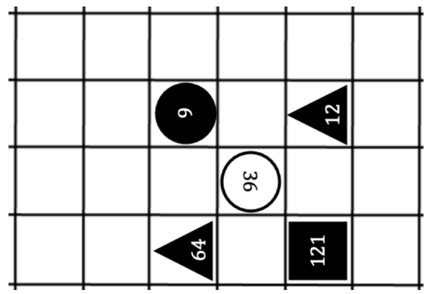
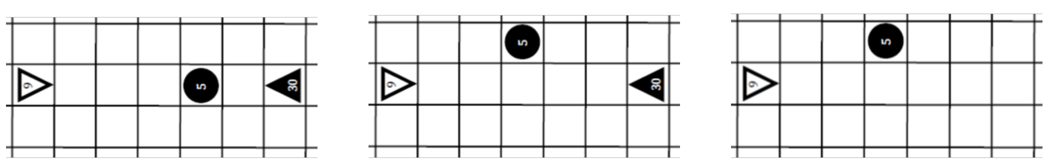


Figure 10: Example of Capture by Surrounding: The black pieces capture the white circle 36 since it is unable to move.

5.7 Innocent Delivery

Innocent delivery occurs when a player makes a move that would allow them to capture an opponent's piece, but without noticing. In these scenarios, if the opponent notices the pieces in some capturing position, without making a move, they can instead capture the initial player's piece. Although they have not moved, this counts as their whole turn and it is back to the initial player's turn.



(a) Setup (b) Black moves (c) White captures

Figure 11: Example of Capture through Innocent Delivery

5.8 Multiple Captures

A player may make a move that results in the capture of multiple pieces. This occurs when by a regular movement, a player's piece is in the position to capture more than one piece through any of the captures described. This is allowed, and the capturing player has successfully captured multiple pieces.

5.9 Piece Return

Though piece return is not a capture, it is an additional type of move that a player can make for their turn. A player may take a piece that they have already captured, flip it over to be their color, and place it anywhere on the half of the board closest to them, the side their pieces started on. This move does take a turn, so once the piece is returned to the board, it is now the opponent's turn. Additionally, the return cannot result in a new capture. The piece can be placed such that a capture can be made on the next move, but the return itself is not a valid capture.

6 The Ending of the Game

Currently, there is no single way in which the game is won. In fact, there are several different ways to end, which simply must be agreed upon by the players at the start of the game. In addition, there are two different classifications of victories, minor and major. In this section, each type of victory within their classifications will be explained.

There are five ways to achieve a minor victory: de bonis, de corpore, de lite, de honore, and de honore litique.

Victory de bonis focuses on point collection. At the beginning of the game, players must agree upon a score that once reached would end the game. Basically, once the values of the piece's captured by a player total the agreed upon score, they have won the game. A standard score would be 100, although this can be adjusted depending on how long players wish to play a single game.

Victory de corpore focuses on the number of pieces. To play this way, players must agree upon a certain number of pieces to collect, and once that quantity of pieces captured has been achieved, that player has won the game.

Victory de lite utilizes a score and a maximum number of digits in order to reach that score. For example, if players agreed to play until someone earned 150 points in four digits, the winner could have collected the pieces 90 and 66 since their sum is 156 and the number of digits between the collected pieces does not exceed four.

For a victory de honore, a player must reach an agreed upon score, but they are limited by a maximum number of pieces. For example, if the score to reach is 100 with a maximum of two pieces, a player may only capture two pieces with values high enough to add to 100 in order to win.

Victory de honore litique combines concepts from victory de lite and victory de honore. A score is agreed upon as well as a maximum number of digits and a maximum number of pieces.

Major victories rely on progressions. There are three different types of major victories: magna, superior, and excellensitissima.

Magna victories are achieved when a player gets three of their pieces to the other half of the board in any of the three progressions: arithmetic, geometric, or harmonic. These pieces must be arranged in a straight line (vertical, horizontal, or diagonal) or as a right angled triangle, though order does not matter.

Superior victories are accomplished when a player gets four of their pieces to the other half of the board, related through progressions. The four pieces must be values such that one subset of three forms a progression and another subset of three forms another progression. These pieces must be arranged in a straight line (vertical, horizontal, or diagonal) or form a rectangle or parallelogram.

Excellentsitissima victories, the most difficult of all, happen when a player gets four of their pieces to the other half of the board and these pieces form all three types of progressions using different subsets of three pieces. There are only three possibilities for this relationship to occur. These pieces must be arranged in a straight line (vertical, horizontal, or diagonal) or form a rectangle or parallelogram.

7 Conclusion

As can be seen from this collection of rules, there is much to be decided in terms of unifying and simplifying game play. In our experience watching real games in Venezuela and the United States of America, the most commonly used capture of pieces is the Capture by Assault. The typical ending of the game is de Bonis (the one that focuses on point collection). The average match takes about 30 to 40 minutes for beginners, but with experience, the playtime reduces drastically to about 10 to 15 minutes. Despite the possibility of a simplification of captures and ending, we believe that completely removing the rest of the captures and victories sacrifices the game's soul. Therefore, while we believe the rules could be simplified down to the Capture by Assault and the Victory de bonis, our hope is that the rest of the rules can remain and be added in as players learn and get more comfortable with the game. We hope to begin the discussion for a clearly defined set of rules and intend to pursue further research of the game.

8 Acknoledgement

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Rié Durnil (rdurnil@zagmail.gonzaga.edu)

Mathematics Department. Gonzaga University.

Tomás Guardia (guardia@gonzaga.edu)

Mathematics Department. Gonzaga University.